

Determination of servomotor parameters of a tripod-based parallel kinematic machine *

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Abstract This paper presents a comprehensive methodology to determine the servomotor parameters of a parallel kinematic machine according to the design specifications in terms of the cutting load, fast feed rate and accelerating capabilities. Due to the characteristics of the parallel format, the singular value decomposition technique is employed to determine the lower and upper bounds of the servomotor parameters such as the rated speed and torque. An approach is also developed to determine the maximum equivalent moment of inertia of the platform and struts reduced onto a servo axis, which therefore allows the accelerating capability of the servo system to be examined in terms of the maximum torque required. The effectiveness of this methodology is exemplified by the application to a tripod-based parallel kinematic machine for high-speed milling.

Keywords: parallel kinematic machine, inverse dynamics, servomotor.

Proper choice of the servomotor parameters is one of the important issues in the design of the numerical control (NC) systems of parallel kinematic machines^[1] (PKMs). This is primarily concerned with the determination of the rated speed and torque of the motor in accordance with the geometrical and inertial parameters of the system, the magnitude of cutting load, specified fast feed rate and accelerating capabilities.

As far as a conventional machine tool is concerned, this may be a trivial task^[2-4]. However, the complex geometry as well as the nonlinear mapping relationship between the Cartesian space and actuator space throughout the workspace implies that a proper choice of servomotor parameters for parallel kinematic machines is by no means an easy task.

It is easy to realize that the inverse dynamics would be the fundamentals in dealing with the issue of this kind. Tremendous and exhaustive work has been carried out over the last decades^[5-8].

In this paper, we present a comprehensive methodology for determining the servomotor parameters of parallel kinematic machines according to the design specifications in terms of the cutting load, fast feed rate and accelerating capability. An example of application is given to determine the

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servomotor parameters of a tripod-based parallel kinematic machine^[9] for high-speed milling.

1 Kinematic analysis

1.1 Inverse kinematics

As shown in Fig. 1, the PKM under consideration consists of a mobile platform and three carriages moving vertically along guide tracks attached to the corresponding columns. In each kinematic chain, the platform and the carriages are spherically jointed by three identical struts having fixed lengths which constitute a set of spatial parallelograms. Three spherical joints on a carriage constitute an equilateral triangle. The one on the top is used to generate an over-constraint that enables the backlash in the joints to be eliminated and thereby the rigidity of the platform to be dramatically increased. The positions of three carriages can be independently manipulated by the corresponding servomotor-leadscrew assemblies, providing the mobile platform with a 3-axis translational moving capability in Cartesian space.

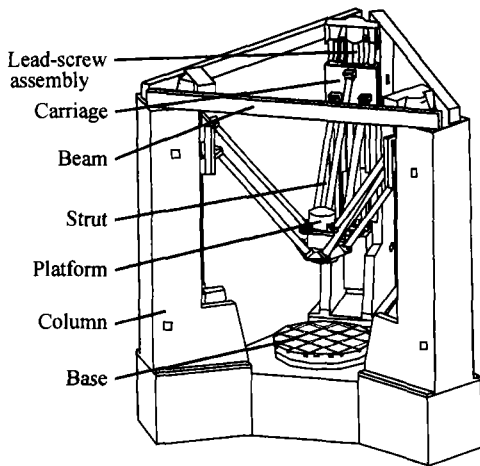


Fig. 1 A Tripod-based PKM.

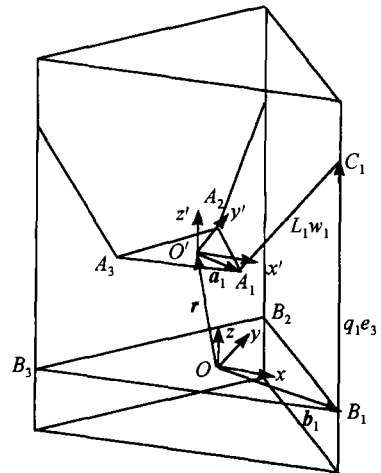


Fig. 2 Kinematic model.

A solution to the inverse kinematics is required for the inverse dynamics. For the PKM under consideration, this problem is primarily concerned with the determination of the positions of three carriages given the dimensions of the parallel mechanism and the position of the platform. Since the motions of three struts associated with a carriage are identical thanks to the parallelogram layout, a simplified model as shown in Fig. 2 can be used for the inverse kinematic analysis. In a Cartesian system, we set the moving frame $Q'-x'y'z'$ to be attached to the platform and the fixed frame $O-xyz$ attached to the base. The frame $Q'-x'y'z'$ is set in such a way that it always keeps parallel to $O-xyz$. The position vector $r = (x \ y \ z)^T$ of the point O' in $O-xyz$ can be expressed as

$$r = b_i - a_i + q_i e_3 + L w_i, \quad \forall i = 1, 2, 3, \tag{1}$$

where $b_i = r_b (\cos \beta_i \ \sin \beta_i \ 0)^T$ and $a_i = r_a (\cos \beta_i \ \sin \beta_i \ 0)^T$ are the position vectors of points B_i and A_i measured in $O-xyz$ and $O'-x'y'z'$; r_a and r_b the radii of the platform and base; $\beta_i = -\pi/$

$6 + 2(i - 1)\pi/3$ is the position angles of points B_i and A_i measured in O - xyz and O' - $x'y'z'$; q_i the displacement of the i th carriage with respect to the point B_i ; w_i and L are the unit vectors of the i th strut axis and its length; $e_3 = (0 \ 0 \ 1)^T$. For a given r , taking the norm on both sides of Eq. (1) gives

$$q_i = r^T e_3 + \sqrt{(r^T e_3)^2 - |r - b_i + a_i|^2 + L^2}, \quad (2)$$

which leads to

$$w_i = (r - b_i + a_i - q_i e_3)/L.$$

1.2 Velocity and acceleration analysis

The representations of velocity and acceleration involve a set of vector manipulations. Differentiating Eq. (1) with respect to time gives

$$v = \dot{q}_i e_3 + L(\omega_i \times w_i), \quad (3)$$

where v denotes the velocity of the reference point O' , \dot{q}_i and ω_i represent the velocity of the i th carriage and the angular velocity of the i th strut, respectively. Taking dot product on both sides of Eq. (1) by w_i leads to

$$\dot{q}_i = J_i^T v.$$

In the matrix form, we have

$$\dot{q} = Jv, \quad (4)$$

where $J = [J_1 \ J_2 \ J_3]^T$, $J_i = \frac{w_i}{w_i^T e_3}$. Taking cross product on both sides of Eq. (2) by w_i will yield

$$\omega_i = \frac{1}{L}(w_i \times (E_3 - e_3 J_i^T))v = J_{\omega i} v. \quad (5)$$

The acceleration of O' can be obtained by differentiating Eq. (3) with respect to time

$$a = \ddot{q}_i e_3 + L(\varepsilon_i \times w_i) + L\omega_i \times (\omega_i \times w_i). \quad (6)$$

Again, taking dot product on both sides of Eq. (6) by w_i leads to the acceleration of the i th carriage

$$\ddot{q}_i = J_i^T a + v^T H_i v.$$

In the matrix form, we have

$$\ddot{q} = Ja + VH\hat{V}, \quad (7)$$

where

$$\ddot{\mathbf{q}} = (\ddot{q}_1 \quad \ddot{q}_2 \quad \ddot{q}_3)^T, \mathbf{H}_i = \frac{1}{L(\mathbf{w}_i^T \mathbf{e}_3)} (\mathbf{E}_3 - \mathbf{e}_3 \mathbf{J}_i^T)^T (\mathbf{E}_3 - \mathbf{e}_3 \mathbf{J}_i^T),$$

$$\mathbf{V} = \text{diag}(\mathbf{v}^T \quad \mathbf{v}^T \quad \mathbf{v}^T), \mathbf{H} = \text{diag}(\mathbf{H}_1 \quad \mathbf{H}_2 \quad \mathbf{H}_3), \hat{\mathbf{V}} = \begin{bmatrix} \mathbf{v} \\ \mathbf{v} \\ \mathbf{v} \end{bmatrix}.$$

The angular acceleration of the i th strut can then be obtained by taking cross product on both sides of Eq. (6)

$$\boldsymbol{\varepsilon}_i = \mathbf{J}_{\omega i} \mathbf{a} - \frac{1}{L} \mathbf{v}^T \mathbf{H}_i \mathbf{v} (\mathbf{w}_i \times \mathbf{e}_3). \quad (8)$$

If the strut is considered as a uniform rod, the velocity and acceleration of its mass center can be represented by

$$\mathbf{v}_i = \frac{1}{2} (\mathbf{E}_3 - \mathbf{e}_3 \mathbf{J}_i^T) \mathbf{v} = \mathbf{J}_{vi} \mathbf{v}. \quad (9)$$

$$\mathbf{a}_i = (\mathbf{J}_i^T \mathbf{a} + \mathbf{v}^T \mathbf{H}_i \mathbf{v}) \mathbf{e}_3 + \frac{L}{2} \left(\mathbf{J}_{\omega i} \mathbf{a} \times \mathbf{w}_i - \frac{1}{L} \mathbf{v}^T \mathbf{H}_i \mathbf{v} (\mathbf{E}_3 - \mathbf{w}_i \mathbf{w}_i^T) \mathbf{e}_3 \right)$$

$$+ \frac{L}{2} (\mathbf{J}_{\omega i} \mathbf{v} \mathbf{v}^T \mathbf{J}_{\omega i}^T - \mathbf{v}^T \mathbf{J}_{\omega i}^T \mathbf{J}_{\omega i} \mathbf{v}) \mathbf{w}_i. \quad (10)$$

2 Evaluation of servomotor parameters

2.1 Determination of the rated speed

Rewrite Eq. (4) such that

$$\mathbf{v} = \mathbf{G} \dot{\mathbf{q}}, \quad \mathbf{G} = \mathbf{J}^{-1}. \quad (11)$$

With the normalization condition $|\dot{\mathbf{q}}| = 1$, the standard eigenvalue problem may be formulated to predict the rated speed of the servomotor required to achieve the specified fast feed rate of the cutting tool

$$(\mathbf{G}^T \mathbf{G} - \sigma_v^2 \mathbf{E}_3) \dot{\mathbf{q}} = 0, \quad |\dot{\mathbf{q}}| = 1, \quad (12)$$

where \mathbf{E}_3 denotes a unit matrix of order 3, σ_v^2 (σ_v is also known as the singular value of matrix \mathbf{G}) and $\dot{\mathbf{q}}$ represent the eigenvalue and eigenvector of $\mathbf{G}^T \mathbf{G}$, respectively. $\sigma_{v\max}$ ($\sigma_{v\min}$) can be interpreted as the maximum (minimum) extreme value of the feed rate of cutting tool when $|\dot{\mathbf{q}}|_{\max(\min)} = 1$. Hence, given the velocity V_C of the carriage (V_C can be specified by the rated speed N (rotating

speed per minute, rpm) and the pitch of the lead-screw(p), i. e. $V_C = Np/60$), the feed rate V_P of the cutting tool can then be determined by the following inequality:

$$\frac{\sigma_{v\min}}{\max(\dot{\mathbf{q}}_{i\min})} V_C \leq V_P \leq \frac{\sigma_{v\max}}{\max(\dot{\mathbf{q}}_{i\max})} V_C, \quad (13)$$

where $\max(\dot{\mathbf{q}}_{i\max})$, $\max(\dot{\mathbf{q}}_{i\min})$ represent the maximum component of eigenvectors associated with $\sigma_{v\max}$, $\sigma_{v\min}$. In the case where the fast feed rate V_P of the cutting tool is specified, the range of the carriage velocity V_C required for the specified V_P can be determined by

$$\frac{\max(\dot{\mathbf{q}}_{i\max})}{\sigma_{v\max}} V_P \leq V_C \leq \frac{\max(\dot{\mathbf{q}}_{i\min})}{\sigma_{v\min}} V_P. \quad (14)$$

Given the lead screw pitch, it is recommended that the mean value of V_C throughout the overall workspace be used to specify the rated speed of the servomotor.

2.2 Inverse dynamics

Inverse dynamics of the system can be formulated using the virtual work principle. Assume that the system is conserved, i. e. with no energy dissipation, the virtual work principle gives

$$\delta \dot{\mathbf{q}}^T \boldsymbol{\tau} = \delta \dot{\mathbf{q}}^T \mathbf{f}_C + \sum_{i=1}^3 (\delta \boldsymbol{\omega}_i^T \mathbf{m}_{S_i} + \delta \mathbf{v}_i^T \mathbf{f}_{S_i}) + \delta \mathbf{v}^T (\mathbf{f}_P + \mathbf{f}_A), \quad (15)$$

where $\mathbf{f}_P = -m_P \mathbf{a}$, the inertial force acting at O' , m_P is the mass of the platform; $\mathbf{f}_{S_i} = -3m_S \mathbf{a}_i$, $\mathbf{m}_{S_i} = -3I_S \boldsymbol{\varepsilon}_i$, the inertial force and moment of the i th group of struts; m_S and I_S , mass and inertia tensor of a single strut; $\mathbf{f}_C = -m_C \ddot{\mathbf{q}}$, the inertial force of the carriage-lead-screw-coupler assembly; $m_C = m'_C + \left(\frac{2\pi}{p}\right)^2 (I_C + I_M)$; m'_C and I_C , the mass and the moment of inertia of the carriage-lead-screw-coupler assembly; I_M , the moment of inertia of the servomotor's rotor; \mathbf{f}_A , the cutting force reduced to the reference point O' ; $\boldsymbol{\tau} = [\tau_1 \quad \tau_2 \quad \tau_3]^T$, the driving force of the carriages.

Substituting Eqs. (4), (5) and (9) in terms of the independent variable $\delta \dot{\mathbf{q}}$ into Eq. (15) leads to

$$\boldsymbol{\tau} = \mathbf{f}_C + \mathbf{G}^T \left(\sum_{i=1}^3 (\mathbf{J}_{\omega_i}^T \mathbf{m}_{S_i} + \mathbf{J}_{v_i}^T \mathbf{f}_{S_i}) + (\mathbf{f}_P + \mathbf{f}_A) \right), \quad (16)$$

which indicates that the driving force provided by the servomotor is a heavily nonlinear function of the feed rate and acceleration of the cutting tool, the geometrical and inertial parameters as well as the configuration of the system.

2.3 Determination of the rated torque

In principle, the criteria that should be followed in the determination of the rated torque of the servomotor can be expressed as follows: (i) the rated torque of the servomotor must be greater than the maximum static load torque caused by the static cutting load and friction; (ii) the moment of inertia of the servomotor must be compatible with that of the system in order to reduce the fluctuation in

torque during the accelerating and decelerating process; (iii) the specified acceleration and deceleration must be obtained.

2.3.1 Determination of the rated torque due to the cutting load. The linear mapping relationship between the static cutting load f_A and the driving force τ of the carriages can be formulated using Eq. (16)

$$\tau = G^T f_A. \quad (17)$$

Hence, given the magnitude F_A of f_A , using a procedure similar to that described in Section 2.1, the static load torque T_S applied on the servomotor shaft can be evaluated by

$$\frac{\rho}{2\pi\eta} \frac{\sigma_{vmin}}{\max(\dot{q}_{imin})} F_A \leq T_S \leq \frac{\rho}{2\pi\eta} \frac{\sigma_{vmax}}{\max(\dot{q}_{imax})} F_A, \quad (18)$$

where η represents the mechanical efficiency of the system. It would be reasonable to take $\eta \approx 0.9 \sim 0.95$ in the case where the low friction components such as rolling guide tracks, lead-screws and the universal joints equipped with ball bearings are used. For the sake of safety, it is strongly recommended that the upper bound of the overall mean value of T_S be adopted as the rated torque of the servomotor.

2.3.2 Examination of the accelerating capability. Only the case where the motor starts up with a linear acceleration mode is considered due to the limitation of space. By neglecting the centrifugal and Coriolis components in Eq. (15), the linear mapping between the acceleration of the carriages and their driving force can be written as

$$\tau = A\ddot{q}, \quad (19)$$

where

$$A = A_C + A_S + A_P,$$

$$A_C = m_C E_3, \quad A_S = G^T \left(J_{wi}^T I_S J_{wi} + m_S J_{vi}^T \left(e_3 J_i^T - \frac{L}{2} \tilde{w}_i \right) J_{wi} \right) G, \quad A_P = m_P G^T G,$$

and A represents the inertial matrix of the system measured in the actuator space. It can be seen that A consists of three components arising from the inertia of the lead-screw assembly, struts and platform, respectively. It is obvious that A_S and A_P are strongly coupled with respect to all the three servo axes due to the parallel format and vary with the change of the system configuration. This is completely different from the conventional machine tools with a serial layout. In order to evaluate the maximum value of the coupled inertia, the orthogonal coordinate transformation $\ddot{q} = \tilde{C}\ddot{q}'$ could be used such that

$$\tau' = C^T \tau = \text{diag}[m_C + \lambda_i] \ddot{q}', \quad (20)$$

where, $\lambda_i > 0, (i = 1, 2, 3)$ represents the i th eigenvalue of matrix $A_S + A_P$. This will enable us to

evaluate the upper bound of the transient inertial torque T_D applied to the motor shaft by setting $m_E = \lambda_{\max}$, the maximum equivalent mass, allocated to each servo axis

$$T_D = (m_C + m_E) \frac{p^2 N_{\max}}{120\pi \eta t_{ac}}, \quad (21)$$

where N_{\max} is the maximum rotating speed per minute (rpm) of the servomotor, t_{ac} , the time interval to reach N_{\max} from rest (normally 3 ~ 4 times of the mechanical time constant of the motor). Note that T_D varies with the system configuration. It is recommended that the upper bound of the overall mean value of T_D be used as an indicator to examine the accelerating capability of the servomotor.

As indicated previously, another criterion in the determination of servomotor parameters is that the moment of inertia of the servomotor I_M must be compatible with that of the system I_L whose upper bound can be expressed as

$$I_L = \left(\frac{p}{2\pi}\right)^2 (m'_C + m_E) + I_C. \quad (22)$$

Hence I_M should fall into $1 < I_M/I_L < 4$ as indicated in Ref. [6]. In addition, attention should be paid to the fact that m_E varies in the workspace, the overall mean value of m_E should therefore be used to evaluate I_L in Equation (22).

The accelerating capability of the cutting tool can also be evaluated as long as the servomotor parameters are available. Substituting Eq. (8) into Eq. (11) leads to a linear mapping between the driving forces of the carriages and the acceleration of the cutting tool

$$a = D\tau, \quad D = GA^{-1}. \quad (23)$$

Similar procedure in the previous sections can be employed to evaluate the range of acceleration A_p by solving the standard eigenvalue problem as follows:

$$(D^T D - \sigma^2 E_3) \tau = 0, \quad |\tau| = 1. \quad (24)$$

This will lead to

$$\frac{2\pi}{p} \frac{\sigma_{\tau \min}}{\max(\tau_{i \min})} T_{\max} \leq A_p \leq \frac{2\pi}{p} \frac{\sigma_{\tau \max}}{\max(\tau_{i \max})} T_{\max}, \quad (25)$$

where T_{\max} represents the maximum output torque of the servomotor, $\max(\tau_{i \max})$ ($\max(\tau_{i \min})$) denotes the maximum component of the eigenvector associated with $\sigma_{\tau \max}$ ($\sigma_{\tau \min}$).

3 Example

As an example of how the method is used, the servomotor parameters of the PKM are predicted with the geometrical and inertial parameters of the system listed in Tables 1 and 2.

Table 1 Dimensional parameters/mm

Radius of platform	Radius of base	Length of strut	Pitch of leading-screw
115	664.13	900	10

Table 2 Inertial parameters

Mass of platform/kg	Mass of strut/kg	Moment inertia of strut/kg·m ²	Mass of carriage/kg	Moment of inertia of leading screw/kg·m ²	Moment of inertia coupler/kg·m ²
77	7.4	0.52	29	10.07×10^{-4}	6×10^{-4}

Figure 3 shows the variation of the carriage speed required to achieve unit feed rate of the cutting tool in the horizontal plane of the workspace of $\Phi 500 \times 400$ mm. It can be seen that the upper and lower bounds of V_C/V_P are $1 \sim 2.02$ and $0.62 \sim 0.86$, respectively. In other words, if $N = 2000$ r/min such that $V_C = 20$ m/min, the upper and lower bounds of the feed rate of the cutting tool are then $23.3 \sim 32.1$ m/min and $9.87 \sim 20$ m/min with the overall mean value of 19.5 m/min.

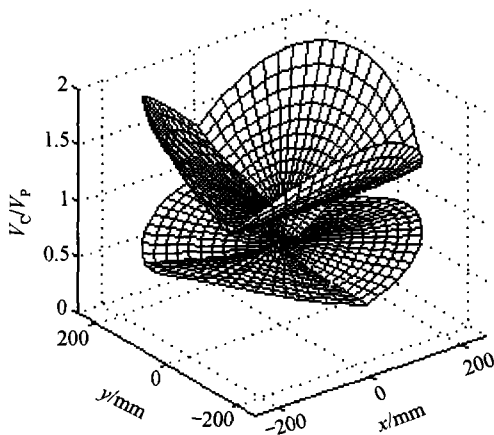


Fig. 3 Lower and upper bounds of the carriage speed required to achieve unit speed rate of the cutting tool.

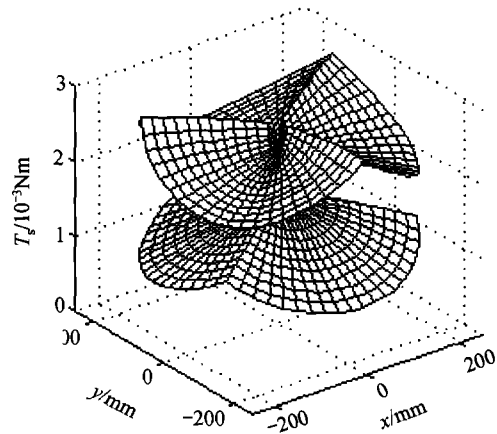


Fig. 4 Upper and lower bounds of the static torque required to balance the unit cutting load.

Figure 4 shows the variation of the static load torque acting on the servomotor shaft due to unit static load applied to the reference point of the platform. It can be seen that the upper and lower bounds of T_S/F_A are $(21 \sim 28) \times 10^{-4}$ and $(8.7 \sim 18) \times 10^{-4}$ (Nm/N). For instance, if the maximum static cutting load is set to be 2000N, then the upper and lower bounds of T_S become $4.2 \sim 5.6$ Nm and $1.74 \sim 3.6$ Nm with the upper bound of the overall mean value of 4.9 Nm. Hence it would be reasonable to select a servomotor having the rated torque of $6 \sim 8$ Nm.

Figure 5 shows the variation of the maximum equivalent mass m_E on each servo axis due to the coupled component of inertia. It can be seen that m_E varies within $107.84 \sim 124.12$ kg and thereby the equivalent moment of inertia becomes $I_E = (2.73 \sim 3.14) \times 10^{-4}$ kg·m². The computational result also shows that the overall mean value of the moment of inertia of the system is 19.75×10^{-4} kg·m², of which 85% comes from the contribution of the carriage-lead-screw-coupler assembly. Hence,

it is reasonable to select a servomotor having the moment of inertia of $I_M = (35 \sim 40) \times 10^{-4} \text{ kg} \cdot \text{m}^2$.

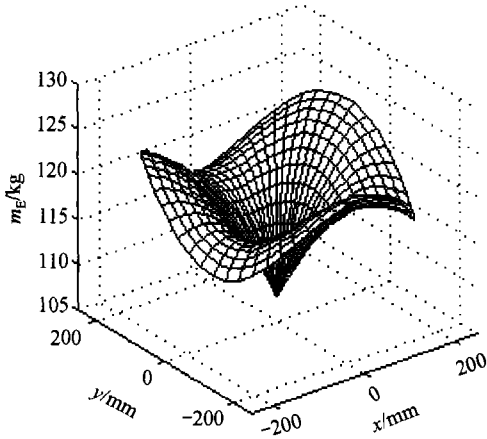


Fig. 5 Maximum value of the coupled inertia of the system allocated to each servo axis.

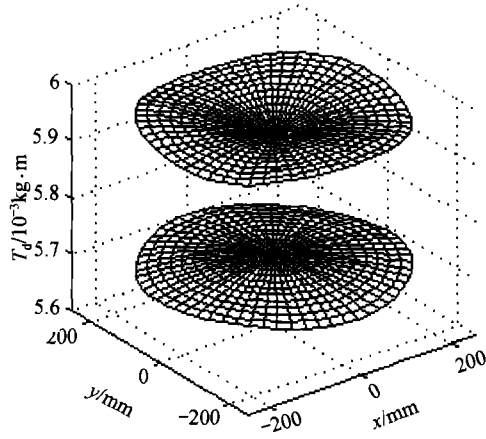


Fig. 6 Lower and upper bounds of torque driving torque required to achieve unit angular acceleration of the motor shaft.

Fig. 6 shows the variation of driving torque needed to achieve unit angular acceleration of the servomotor shaft with $I_M = 40 \times 10^{-4} \text{ kg} \cdot \text{m}^2$. It can be seen that the fluctuation in torque is very small thanks to the large proportion of the non-coupled inertia of the system. Given $t_{ac} = 90 \text{ ms}$ and $\eta = 0.9$, the computational results also show that the driving torque of 15.4 Nm is required for the motor to reach its rated speed of 2000 r/min from the rest, and 24.62 Nm to reach its maximum rate of 3200 r/min . Hence, a servomotor with the maximum output torque of 30 Nm is required.

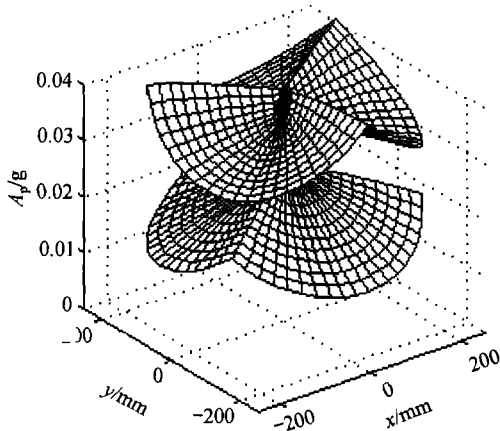


Fig. 7 Upper and lower bounds of the acceleration of the cutting tool that can be achieved by unit driving torque of the servomotor.

Figure 7 shows the variation of the accelerating capability of the cutting tool acquired by unit driving torque. It can be seen that the upper and lower bounds of A_p/T_D are $0.032 \sim 0.044 \text{ g/Nm}$ and $0.014 \sim 0.028 \text{ g/Nm}$ with its overall mean value of 0.030 g/Nm . For instance, if the maximum output torque of the servomotor is 30 Nm , then it can be expected that the cutting tool possesses an accelerating capability of 0.9 g .

4 Conclusions

A systematic method has been developed in this paper for the prediction of servomotor parameters of a tripod based parallel kinematic machine. The results can be summarized as follows.

(i) Given the fast feed rate of the cutting tool, the upper and lower bounds of the carriage can be evaluated and *vice versa* by using the singular de-

composition technique. The overall mean value can be used to determine the rated speed of the servomotor.

(ii) Given the cutting load, the upper and lower bounds of the load torque applied to the motor shaft can be evaluated by the same technique. The upper bound of the overall mean value can serve as the rated torque of the servomotor.

(iii) Due to the parallel format, the inertia of the struts and platform reduced onto the servo axes is coupled but their contributions are rather small compared with the non-coupled component. Consequently, the torque fluctuation due to this coupling is very small thanks to the externally arranged actuator layout.

(iv) Given the maximum output torque of the servomotor, the singular decomposition technique can also be used to evaluate the acceleration capability of the cutting tool.

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